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Jet rates at small x to single-logarithmic accuracy*

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Abstract: We present predictions of jet rates in deep inelastic scattering at small x to leading-logarithmic order in x, including all sub-leading logarithms of $Q^2/\mu_{\rm R}^2$ where $\mu_{\rm R}$ is the transverse momentum scale at which jets are resolved. We give explicit results for up to three jets, and a perturbative expansion for multi-jet rates and jet multiplicities.

Keywords: Deep Inelastic Scattering, QCD, Jets.

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Contents

1. Introduction

The summation of logarithms of $1/x$ in deep inelastic structure functions at small values of Bjorken x leads to the Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation [1, 2], which in the leading approximation sums terms of order $[\alpha_{\rm S}\ln(1/x)]^n$. Recently the next-to-leading terms have also been computed [3, 4].

Since the dynamics of the small-x region is supposed to be different from that at higher x in several respects, it is important to make and test predictions of a wide range of observables in this region. In recent papers [5, 6] predictions were presented for the rates of emission of fixed numbers of 'resolved' final-state gluons, together with any number of unresolvable ones. Here 'resolved' means having a transverse momentum larger than some fixed value $\mu_{\rm R}$. The predictions were valid in the double-logarithmic (DL) approximation, i.e. retaining only terms of the form $[\alpha_{\rm S}\ln(1/x)\ln(Q^2/\mu_{\rm R}^2)]^n$. In this approximation, each resolved gluon can be equated to a single jet, since to resolve it into more than one jet would cost extra powers of $\alpha_{\rm S}$ with no corresponding powers of $\ln(1/x)$.

The present paper extends the work of ref. [6] to include terms with fewer powers of $\ln(Q^2/\mu_{\rm R}^2)$, i.e. those of the form $[\alpha_{\rm S}\ln(1/x)]^n\,[\ln(Q^2/\mu_{\rm R}^2)]^m$ where $0 < m < n$, which we refer to as single-logarithmic (SL) corrections. The fact that we still demand a factor of $\ln(1/x)$ with each power of α_S means that the identification of resolved gluons with jets[1] remains valid.

There are two alternative methods for the calculation of final-state properties at small x: the original multi-Regge BFKL method and the CCFM [7, 8, 9] approach, which takes account of the coherence of soft gluon emission. It has been shown, at the DL level in refs. [5, 6] and now at the SL level [10], that the two methods are equivalent for the observables considered here. We therefore adopt the BFKL approach, which is calculationally simpler.

The paper is organized as follows. In sect. 2 we recall the BFKL formalism and the predicted behaviour of the gluon structure function at small x. In sect. 3 we first compute the single-jet rate to SL accuracy and show that the only modification to the DL result comes from the SL corrections to the anomalous dimension. Next, in subsect. 3.2, we calculate the SL corrections to the two-jet rate, first in the form of a numerical integral and then as a perturbation series. In subsect. 3.3 we apply the same methods to the three-jet rate. In sect. 4 we derive the SL perturbative expansion of the generating function for the multi-jet rates, and use this to obtain the corresponding expansions for the mean jet multiplicity and its dispersion. Our conclusions are presented in sect. 5. A useful class of integrals is evaluated to SL accuracy in the appendix.

2. BFKL formalism

We start from the unintegrated structure function of a single gluon, $f(x, k^2, \mu^2)$, which in the exclusive form of the BFKL approach satisfies the equation

$$f(x,k^2,\mu^2) = \delta(1-x)\,\delta^2(k) + \bar{\alpha}_S \int_{\mu^2} \frac{dq^2}{q^2}\frac{d\phi}{2\pi}\frac{dz}{z^2}\Delta(z,k^2,\mu^2)\, f\left(\frac{x}{z}, |q+k|^2, \mu^2\right). \quad (2.1)$$

Here $\bar{\alpha}_S = 3\alpha_S/\pi$, k is the (2-vector) transverse momentum of the gluon probed in the deep inelastic scattering, q is that of an emitted gluon, ϕ is the azimuthal angle giving the direction of q, μ is a collinear cutoff and Δ is the Regge form factor

$$\Delta(z,k^2,\mu^2) = \exp\left(-\bar{\alpha}_S \ln\frac{1}{z}\ln\frac{k^2}{\mu^2}\right). \quad (2.2)$$

To carry out the z integration it is convenient to use a Mellin representation,

$$f_\omega(k^2,\mu^2) = \int_0^1 dx\, x^\omega f(x,k^2,\mu^2)\,, \quad (2.3)$$

[1] When counting jets in deep inelastic lepton scattering, we always omit final-state hadrons that originate from the quark-antiquark pair which couples the gluon to the virtual photon.

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with inverse

$$f(x, k^2, \mu^2) = \frac{1}{2\pi i} \int_C d\omega\, x^{-\omega-1} f_\omega(k^2, \mu^2)\,, \tag{2.4}$$

where the contour C is parallel to the imaginary axis and to the right of all singularities of the integrand. This gives

$$f_\omega(k^2, \mu^2) = \delta^2(k) + H_\omega(k^2, \mu^2) \int_{\mu^2} \frac{dq^2}{q^2} \frac{d\phi}{2\pi} f_\omega(|q+k|^2, \mu^2)\,, \tag{2.5}$$

where

$$H_\omega(k^2, \mu^2) = \frac{\bar{\alpha}_{\rm S}}{\omega + \bar{\alpha}_{\rm S} \ln(k^2/\mu^2)}\,. \tag{2.6}$$

To solve eq. (2.5) one can make use of the relation, derived in the appendix,

$$\int_{\mu^2}^{Q^2} \frac{dq^2}{q^2} \frac{d\phi}{2\pi} f(|q+k|^2) = \int_{\mu^2}^{Q^2} \frac{dq^2}{q^2} f(\max\{q^2, k^2\}) + 2 \sum_{m=1}^{\infty} \zeta(2m+1) \left(k^2 \frac{\partial}{\partial k^2}\right)^{2m} f(k^2)\,, \tag{2.7}$$

which is valid for $\mu^2 < k^2 < Q^2$ to logarithmic accuracy, i.e. neglecting terms suppressed by powers of μ^2/k^2 or k^2/Q^2. Then for $k^2 > \mu^2$ the solution is of the form

$$f_\omega(k^2, \mu^2) = \frac{\gamma}{\pi k^2} \left(\frac{k^2}{\mu^2}\right)^\gamma\,, \tag{2.8}$$

where

$$1 = H_\omega(k^2, \mu^2) \left(\ln \frac{k^2}{\mu^2} - \frac{1}{\gamma - 1} + 2 \sum_{m=1}^{\infty} \zeta(2m+1)(\gamma-1)^{2m}\right) \tag{2.9}$$

and hence

$$\omega = -\bar{\alpha}_{\rm S}\left[2\gamma_{\rm E} + \psi(\gamma) + \psi(1-\gamma)\right]\,, \tag{2.10}$$

ψ being the digamma function and $\gamma_{\rm E} = -\psi(1)$ the Euler constant. The solution of eq. (2.10) is $\gamma = \gamma_{\rm L}(\bar{\alpha}_{\rm S}/\omega)$, the Lipatov anomalous dimension:

$$\gamma_{\rm L}(\bar{\alpha}_{\rm S}/\omega) = \frac{\bar{\alpha}_{\rm S}}{\omega} + 2\zeta(3)\left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^4 + 2\zeta(5)\left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^6 + 12[\zeta(3)]^2 \left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^7 + \cdots\,. \tag{2.11}$$

The integrated gluon structure function at scale Q^2 is then given by

$$F_\omega(Q^2, \mu^2) = 1 + \pi \int_{\mu^2}^{Q^2} dk^2\, f_\omega(k^2, \mu^2) = \left(\frac{Q^2}{\mu^2}\right)^{\gamma_{\rm L}(\bar{\alpha}_{\rm S}/\omega)}\,. \tag{2.12}$$

Since we are interested in final states, we shall need to decompose the structure function into terms corresponding to different numbers of emitted gluons. The contribution to $f(x, k^2, \mu^2)$ from emission of n gluons is obtained by iteration of eq. (2.1):

$$f^{(n)}(x, k^2, \mu^2) = \prod_{i=1}^{n} \int_{\mu^2} \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} \frac{dz_i}{z_i} \bar{\alpha}_{\rm S} \Delta(z_i, k_i^2, \mu^2) \delta(x - x_n) \delta^2(k - k_n)\,, \tag{2.13}$$

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where

$$x_i = \prod_{l=1}^{i} z_l \,, \qquad k_i = -\sum_{l=1}^{i} q_l \,. \tag{2.14}$$

The contribution to the structure function at scale Q is then obtained by integrating over all $\mu^2 < q_i^2 < Q^2$:

$$F^{(n)}(x, Q^2, \mu^2) = \prod_{i=1}^{n} \int_{\mu^2}^{Q^2} \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} \frac{dz_i}{z_i} \bar{\alpha}_{\mathrm{S}} \Delta(z_i, k_i^2, \mu^2) \delta(x - x_n) \,, \tag{2.15}$$

or in terms of the Mellin transform

$$F_\omega^{(n)}(Q^2, \mu^2) = \prod_{i=1}^{n} \int_{\mu^2}^{Q^2} \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} H_\omega(k_i^2, \mu^2) \,. \tag{2.16}$$

3. Jet rates

3.1 Single-jet rate

Consider first the effect of requiring one emitted gluon, say the jth, to have $q_j^2 > \mu_{\mathrm{R}}^2$ while all the others have $q_i^2 < \mu_{\mathrm{R}}^2$. This defines the contribution of one resolved gluon plus $n-1$ unresolved, $F^{(n,1\,\mathrm{jet})}$:

$$F_\omega^{(n,1\,\mathrm{jet})}(Q^2, \mu_{\mathrm{R}}^2, \mu^2) = \sum_{j=1}^{n} \int_{\mu_{\mathrm{R}}^2}^{Q^2} \frac{dq_j^2}{q_j^2} \frac{d\phi_j}{2\pi} H_\omega(k_j^2, \mu^2) \prod_{i \neq j}^{n} \int_{\mu^2}^{\mu_{\mathrm{R}}^2} \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} H_\omega(k_i^2, \mu^2) \,. \tag{3.1}$$

Notice that for $i < j$ the contribution is identical to the $(j-1)$-gluon contribution to the structure function evaluated at $Q^2 = \mu_{\mathrm{R}}^2$. On the other hand for $i > j$ we have $k_i^2 \simeq q_j^2 > \mu_{\mathrm{R}}^2$. As shown in the appendix, when $q_j^2 > \mu_{\mathrm{R}}^2$ we can write for any function f, to logarithmic accuracy,

$$\int_{\mu^2}^{\mu_{\mathrm{R}}^2} \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} f(|q_i + q_j|^2) = \ln\left(\frac{\mu_{\mathrm{R}}^2}{\mu^2}\right) f(q_j^2) \,. \tag{3.2}$$

Thus the q_i integrations for $i > j$ become trivial and

$$F_\omega^{(n,1\,\mathrm{jet})}(Q^2, \mu_{\mathrm{R}}^2, \mu^2) = \frac{1}{S} \sum_{j=1}^{n} F_\omega^{(j-1)}(\mu_{\mathrm{R}}^2, \mu^2) \int_{\mu_{\mathrm{R}}^2}^{Q^2} \frac{dq_j^2}{q_j^2} \left[S\, H_\omega(q_j^2, \mu^2) \right]^{n-j+1} \,, \tag{3.3}$$

where we define[2]

$$S = \ln\left(\frac{\mu_{\mathrm{R}}^2}{\mu^2}\right) \,, \qquad T = \ln\left(\frac{Q^2}{\mu_{\mathrm{R}}^2}\right) \,. \tag{3.4}$$

[2] Note that we define S and T differently from Refs. [5, 6] (twice as large) in order to simplify expressions for the single-logarithmic terms.

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Summing over all j and n gives the total one-jet contribution,

$$\begin{aligned} F_\omega^{(1\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2,\mu^2) &= F_\omega(\mu_{\mathrm{R}}^2,\mu^2)\int_{\mu_{\mathrm{R}}^2}^{Q^2}\frac{dq_j^2}{q_j^2}H_\omega(q_j^2,\mu_{\mathrm{R}}^2) \\ &= \exp\left[\gamma_{\mathrm{L}}\left(\bar{\alpha}_{\mathrm{S}}/\omega\right)S\right]\,G_\omega^{(1)}(T)\,, \end{aligned} \tag{3.5}$$

where

$$G_\omega^{(1)}(T) = \ln\left(1+\frac{\bar{\alpha}_{\mathrm{S}}}{\omega}T\right). \tag{3.6}$$

Notice that the collinear-divergent part (the S-dependence) factorizes, and the fraction of events with one jet is given by the cutoff-independent function

$$R_\omega^{(1\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2) = \frac{F_\omega^{(1\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2,\mu^2)}{F_\omega(Q^2,\mu^2)} = \exp\left[-\gamma_{\mathrm{L}}\left(\bar{\alpha}_{\mathrm{S}}/\omega\right)T\right]\,G_\omega^{(1)}(T)\,. \tag{3.7}$$

Thus in the case of the single-jet rate, the only subleading logarithms are those generated by the presence of the full Lipatov anomalous dimension (2.11) in eq. (3.7).

To obtain the jet cross section as a function of x, we note that eq. (3.5) implies that

$$F_\omega^{(1\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2) = F_\omega(\mu_{\mathrm{R}}^2)\,G_\omega^{(1)}(T)\,, \tag{3.8}$$

where we have used the factorization property to replace the cutoff-dependent gluon structure function $F_\omega(\mu_{\mathrm{R}}^2,\mu^2)$ by the measured structure function of the target hadron at scale μ_{R}^2, $F_\omega(\mu_{\mathrm{R}}^2)$. It follows that the single-jet contribution as a function of x is given by the convolution

$$F^{(1\,\mathrm{jet})}(x,Q^2,\mu_{\mathrm{R}}^2) = F(x,\mu_{\mathrm{R}}^2)\otimes G^{(1)}(x,T) \equiv \int_x^1\frac{dz}{z}F(z,\mu_{\mathrm{R}}^2)\otimes G^{(1)}\left(\frac{x}{z},T\right)\,, \tag{3.9}$$

where the inverse Mellin transformation (2.4) applied to eq. (3.6) gives

$$G^{(1)}(x,T) = \frac{1}{2\pi i}\int_C d\omega\,x^{-\omega-1}\,G_\omega^{(1)}(T) = \frac{1-x^{\bar{\alpha}_{\mathrm{S}}T}}{x\ln\left(\frac{1}{x}\right)}\,. \tag{3.10}$$

3.2 Two-jet rate

Now suppose we resolve two gluons j, j' ($j<j'$) with transverse momenta $q_j^2, q_{j'}^2 > \mu_{\mathrm{R}}^2$. In place of eq. (3.5) we have

$$F_\omega^{(2\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2,\mu^2) = F_\omega(\mu_{\mathrm{R}}^2,\mu^2)\int_{\mu_{\mathrm{R}}^2}^{Q^2}\frac{dq_j^2}{q_j^2}H_\omega(q_j^2,\mu_{\mathrm{R}}^2)\int_{\mu_{\mathrm{R}}^2}^{Q^2}\frac{dq_{j'}^2}{q_{j'}^2}\frac{d\phi_{j'}}{2\pi}K_\omega(|q_j+q_{j'}|^2,\mu_{\mathrm{R}}^2)\,, \tag{3.11}$$

where, defining the dijet transverse momentum $q_J = q_j + q_{j'}$,

$$K_\omega(q_J^2,\mu_{\mathrm{R}}^2) = H_\omega(q_J^2,\mu^2)\left(1+\sum_{n=j'+1}^{\infty}\prod_{i=j'+1}^{n}\int_{\mu^2}^{\mu_{\mathrm{R}}^2}\frac{dq_i^2}{q_i^2}\frac{d\phi_i}{2\pi}H_\omega(k_i^2,\mu^2)\right). \tag{3.12}$$

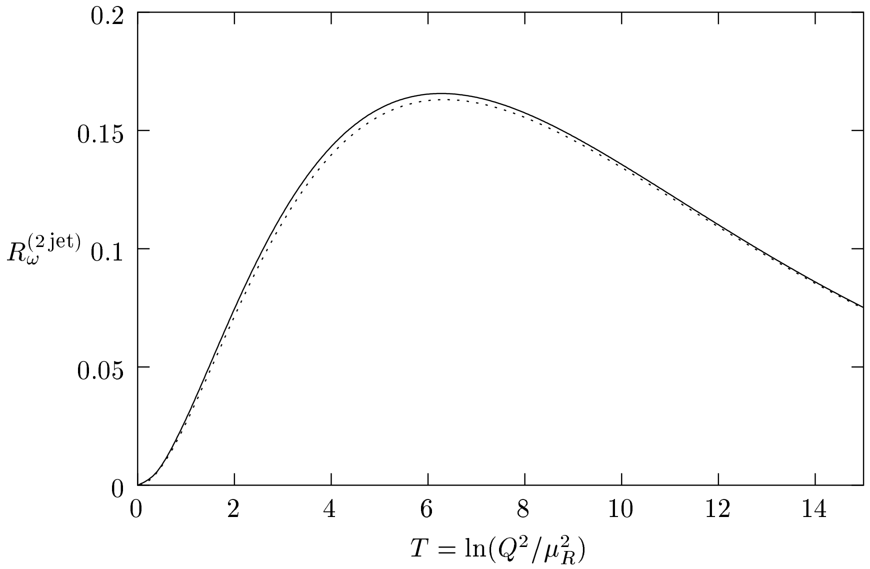

Figure 1: Two-jet rate for $\bar{\alpha}_{\mathrm{S}}/\omega = 0.2$. Dashed: double-log approximation. Solid: with single-log corrections.

When $q_J^2 > \mu_{\mathrm{R}}^2$ we can safely set $k_i^2 = q_J^2$ for $i > j'$, to obtain

$$K_\omega(q_J^2, \mu_{\mathrm{R}}^2) = H_\omega(q_J^2, \mu_{\mathrm{R}}^2)\,. \tag{3.13}$$

However, this cannot be correct for $q_J^2 < \mu_{\mathrm{R}}^2$, because $K_\omega(q_J^2, \mu_{\mathrm{R}}^2)$ would then be infinite at $q_J^2 = \mu_{\mathrm{R}}^2 \exp(-\omega/\bar{\alpha}_{\mathrm{S}})$. Since $K_\omega(q_J^2, \mu_{\mathrm{R}}^2)$ must be independent of the cutoff μ, we can evaluate it for $q_J^2 < \mu_{\mathrm{R}}^2$ by setting $\mu^2 = q_J^2$, which gives

$$K_\omega(q_J^2, \mu_{\mathrm{R}}^2) = \frac{\bar{\alpha}_{\mathrm{S}}}{\omega} F_\omega(\mu_{\mathrm{R}}^2, q_J^2) = \frac{\bar{\alpha}_{\mathrm{S}}}{\omega} \left(\frac{\mu_{\mathrm{R}}^2}{q_J^2}\right)^{\gamma_{\mathrm{L}}(\bar{\alpha}_{\mathrm{S}}/\omega)}\,. \tag{3.14}$$

Thus we can define the continuous function

$$K_\omega(q_J^2, \mu_{\mathrm{R}}^2) = H_\omega(q_J^2, \mu_{\mathrm{R}}^2)\theta(q_J^2 - \mu_{\mathrm{R}}^2) + \frac{\bar{\alpha}_{\mathrm{S}}}{\omega} F_\omega(\mu_{\mathrm{R}}^2, q_J^2)\theta(\mu_{\mathrm{R}}^2 - q_J^2)\,. \tag{3.15}$$

The two-jet rate is then given by

$$R_\omega^{(2\,\mathrm{jet})}(Q^2, \mu_{\mathrm{R}}^2) = \frac{F_\omega^{(2\,\mathrm{jet})}(Q^2, \mu_{\mathrm{R}}^2, \mu^2)}{F_\omega(Q^2, \mu^2)} = \exp\left[-\gamma_{\mathrm{L}}\left(\bar{\alpha}_{\mathrm{S}}/\omega\right) T\right]\, G_\omega^{(2)}(T)\,, \tag{3.16}$$

where

$$G_\omega^{(2)}(T) = \int_{\mu_{\mathrm{R}}^2}^{Q^2} \frac{dq_j^2}{q_j^2} \frac{dq_{j'}^2}{q_{j'}^2} \frac{d\phi_{j'}}{2\pi} K_\omega(q_j^2, \mu_{\mathrm{R}}^2)\, K_\omega(|q_j + q_{j'}|^2, \mu_{\mathrm{R}}^2)\,. \tag{3.17}$$

The integrations in eq. (3.17) can be performed numerically, without encountering any non-integrable divergences or discontinuities in the integrand. The resulting two-jet rate is shown by the solid curve in fig. 1 for a relatively small value of $\bar{\alpha}_{\mathrm{S}}/\omega$

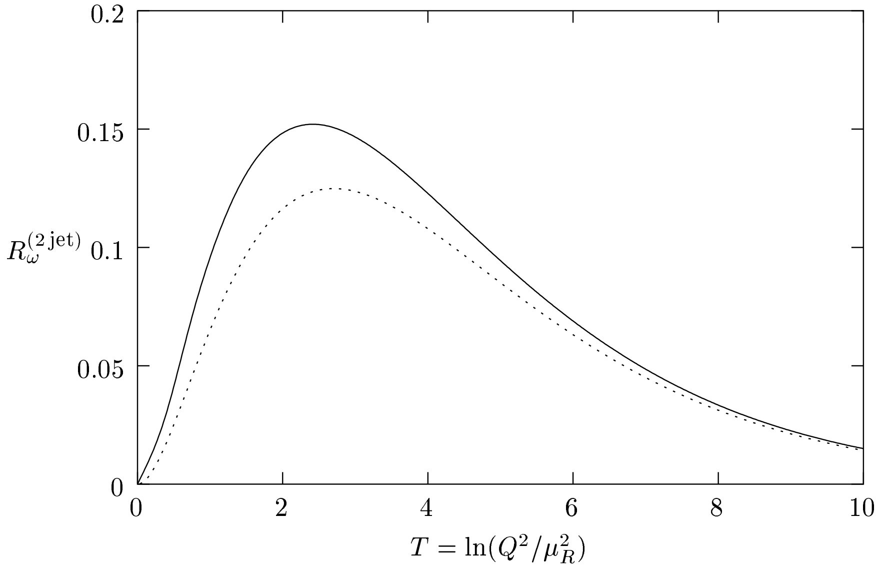

Figure 2: Two-jet rate for $\bar{\alpha}_{\rm S}/\omega = 0.4$. Dashed: double-log approximation. Solid: with single-log corrections.

(0.2), and for a larger value (0.4) in fig. 2. The dashed curves show the result of using the DL prediction for $G_\omega^{(2)}$ in eq. (3.16), i.e.

$$R_\omega^{(2\,\mathrm{jet})}(Q^2,\mu_{\rm R}^2) \simeq \exp\left[-\gamma_{\rm L}\left(\bar{\alpha}_{\rm S}/\omega\right) T\right]\, G_\omega^{(2,\mathrm{DL})}(T)\,, \tag{3.18}$$

where [6]

$$G_\omega^{(2,\mathrm{DL})}(T) = \frac{1}{2}\ln^2\left(1+\frac{\bar{\alpha}_{\rm S}}{\omega}T\right) + \ln\left(1+\frac{\bar{\alpha}_{\rm S}}{\omega}T\right) - \frac{\bar{\alpha}_{\rm S}}{\omega}T\left(1+\frac{\bar{\alpha}_{\rm S}}{\omega}T\right)^{-1}. \tag{3.19}$$

We see that for $\bar{\alpha}_{\rm S}/\omega = 0.2$ the single-logarithmic correction is small, while for the larger value it is substantial.

Next we consider the perturbative expansion of the two-jet rate. We can use eq. (3.15) and the results in the appendix to obtain

$$\int_{\mu_{\rm R}^2}^{Q^2}\frac{dq_{j'}^2}{q_{j'}^2}\frac{d\phi_{j'}}{2\pi}K_\omega(|q_j+q_{j'}|^2,\mu_{\rm R}^2) = \int_{\mu_{\rm R}^2}^{Q^2}\frac{dq_{j'}^2}{q_{j'}^2}H_\omega(\max\{q_j^2,q_{j'}^2\},\mu_{\rm R}^2) + 2\sum_{m=1}^{\infty}(2m)!\,\zeta(2m+1)\left[H_\omega(q_j^2,\mu_{\rm R}^2)\right]^{2m+1}. \tag{3.20}$$

Substituting in eq. (3.17) we find

$$G_\omega^{(2)}(T) = G_\omega^{(2,\mathrm{DL})}(T) + G_\omega^{(2,\mathrm{SL})}(T)\,, \tag{3.21}$$

where $G_\omega^{(2,\mathrm{DL})}$ is the double-logarithmic result (3.19) and the single-logarithmic correction is

$$G_\omega^{(2,\mathrm{SL})}(T) = 2\sum_{m=1}^{\infty}\frac{(2m)!}{2m+1}\zeta(2m+1)\left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^{2m+1}\left[1-\left(1+\frac{\bar{\alpha}_{\rm S}}{\omega}T\right)^{-2m-1}\right]. \tag{3.22}$$

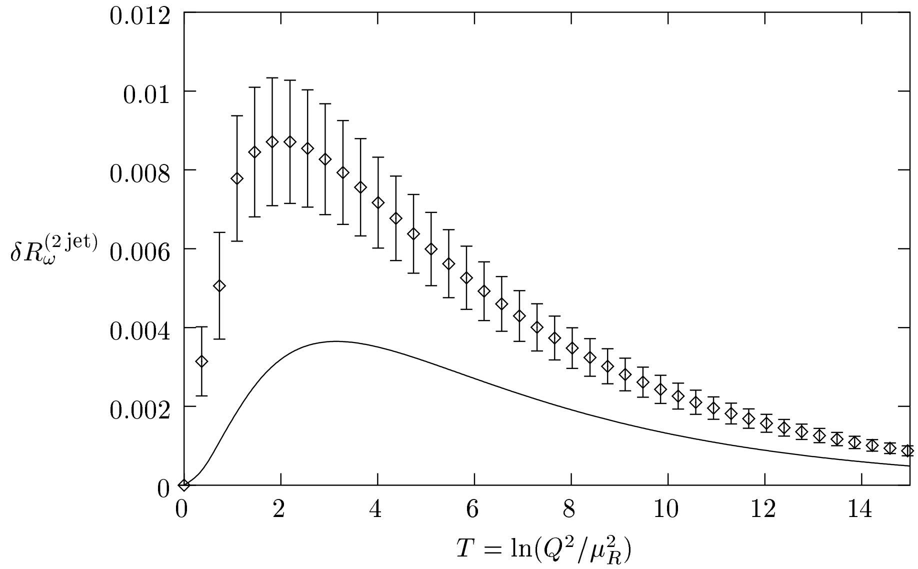

Figure 3: Solid: single-log correction to two-jet rate for $\bar{\alpha}_{\mathrm{S}}/\omega = 0.2$. Points: estimate from asymptotic expansion; 'error bars' indicate the smallest term in the expansion.

Note that the series in eq. (3.22) is strongly divergent for any value of $\bar{\alpha}_{\mathrm{S}}/\omega$. This is due to the singularity of $H_\omega(q_J^2, \mu_{\mathrm{R}}^2)$ at $q_J^2 = \mu_{\mathrm{R}}^2 \exp(-\omega/\bar{\alpha}_{\mathrm{S}}) < \mu_{\mathrm{R}}^2$. The change in $K_\omega(q_J^2, \mu_{\mathrm{R}}^2)$ when $q_J^2 < \mu_{\mathrm{R}}^2$ (see eq. (3.15)) removes the singularity from the integrand in eq. (3.17) but this does not affect the region of convergence of the perturbation series. The situation is analogous to the way in which the running of the QCD coupling $\alpha_{\mathrm{S}}(q^2)$ at low values of q^2 produces infrared renormalons [11]: the Landau singularity in the perturbative expression for $\alpha_{\mathrm{S}}(q^2)$ leads to a factorial divergence of perturbative expansions with respect to $\alpha_{\mathrm{S}}(Q^2)$, where Q^2 is fixed and large, even if we remove the Landau singularity by making a non-perturbative modification of $\alpha_{\mathrm{S}}(q^2)$ at low q^2 [12].

In the case of eq. (3.20), the correction arising from the more careful treatment of the region $q_J^2 < \mu_{\mathrm{R}}^2$, corresponding to the second term in eq. (3.15), would be

$$\begin{aligned}\delta \int_{\mu_{\mathrm{R}}^2}^{Q^2} \frac{dq_{j'}^2}{q_{j'}^2} \frac{d\phi_{j'}}{2\pi} K_\omega(q_J^2, \mu_{\mathrm{R}}^2) &\simeq \frac{\bar{\alpha}_{\mathrm{S}}}{\omega} \int_0^{\mu_{\mathrm{R}}^2} \frac{dq_J^2}{q_j^2} \left[\left(\frac{\mu_{\mathrm{R}}^2}{q_J^2} \right)^{\gamma_{\mathrm{L}}(\bar{\alpha}_{\mathrm{S}}/\omega)} - \sum_{m=0}^{\infty} \left(-\frac{\bar{\alpha}_{\mathrm{S}}}{\omega} \ln \frac{q_J^2}{\mu_{\mathrm{R}}^2} \right)^m \right] \\ &= \frac{\bar{\alpha}_{\mathrm{S}}}{\omega} \frac{\mu_{\mathrm{R}}^2}{q_j^2} \left[\frac{1}{1 - \gamma_{\mathrm{L}}(\bar{\alpha}_{\mathrm{S}}/\omega)} - \sum_{m=0}^{\infty} m! \left(\frac{\bar{\alpha}_{\mathrm{S}}}{\omega} \right)^m \right] . \end{aligned} \tag{3.23}$$

This does indeed contain a factorially divergent series, but, owing to the overall factor of $1/q_j^2$, it does not contribute any logarithms of Q^2/μ_{R}^2 upon substitution in eq. (3.17). Therefore a more accurate treatment of the region $q_J^2 < \mu_{\mathrm{R}}^2$, although necessary to evaluate the integrals, does not affect the two-jet rate to SL precision.

If we interpret the series in eq. (3.22) as an asymptotic expansion, then the partial sum truncated after the smallest term represents an estimate of the total SL correction, with an uncertainty of the order of the smallest term. This estimate is

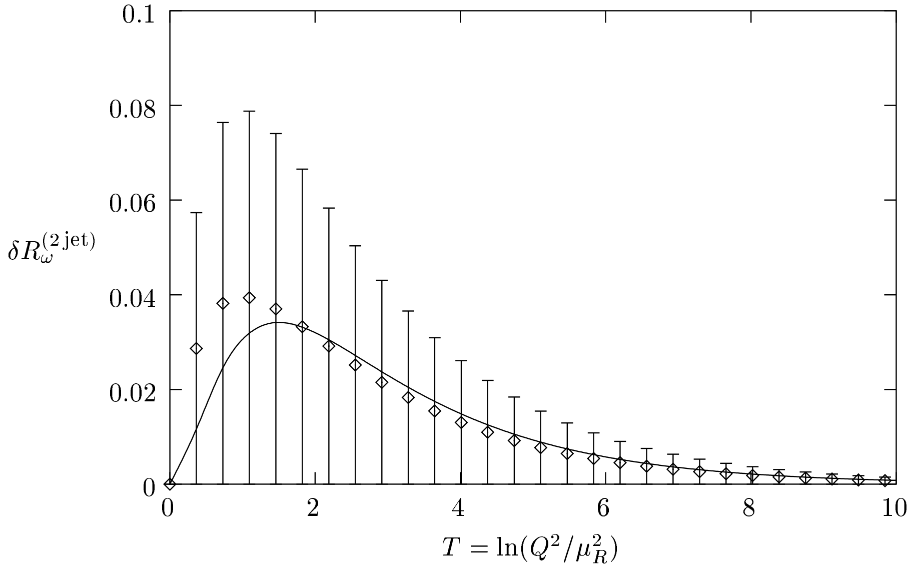

Figure 4: Solid: single-log correction to two-jet rate for $\bar{\alpha}_S/\omega = 0.4$. Points: estimate from asymptotic expansion; 'error bars' indicate the smallest term in the expansion.

shown by the points in figs. 3 and 4, with the uncertainty represented by the error bars. We see that estimates from the series are of the same order of magnitude as the numerical results, but the discrepancy may be several times the expected uncertainty. At the smaller value of $\bar{\alpha}_S/\omega$, the SL correction is relatively small (c.f. fig. 1) and the discrepancy is not so important. For the larger value of $\bar{\alpha}_S/\omega$, the estimate is better than expected, but the correction and the uncertainty are both large.

To deduce the two-jet rate as a function of x, we can proceed as in eq. (3.9), writing

$$F^{(2\,\mathrm{jet})}(x, Q^2, \mu_R^2) = F(x, \mu_R^2) \otimes G^{(2)}(x, T)\,, \tag{3.24}$$

where

$$G^{(2)}(x, T) = \frac{1}{2\pi i}\int_C d\omega\, x^{-\omega-1}\, G_\omega^{(2)}(T)\,. \tag{3.25}$$

For the DL contribution we find from eq. (3.19) that

$$G^{(2,\mathrm{DL})}(x, T) = \frac{1}{x\ln(1/x)}\,\mathcal{G}^{(2,\mathrm{DL})}\left[\bar{\alpha}_S T \ln\left(\frac{1}{x}\right)\right], \tag{3.26}$$

where

$$\mathcal{G}^{(2,\mathrm{DL})}[z] = E_1(z) + \ln z + \gamma_E + 1 + e^{-z}\left[E_1(-z) + \ln z + \gamma_E - 1 - z\right], \tag{3.27}$$

$E_1(z)$ being the exponential integral function

$$E_1(z) = \int_z^\infty dt\frac{e^{-t}}{t}\,, \tag{3.28}$$

interpreted as a principal-value integral when $z < 0$.

The divergence of the series for the SL correction in ω-space, eq. (3.22), is cured when one makes the inverse Mellin transformation to x-space, because the factorial coefficients are cancelled:

$$\frac{1}{2\pi i}\int_C d\omega\, x^{-\omega-1}\left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^{2m+1} = \frac{\bar{\alpha}_{\rm S}}{x}\frac{\left[\bar{\alpha}_{\rm S}\ln\left(\frac{1}{x}\right)\right]^{2m}}{(2m)!}\,. \tag{3.29}$$

Thus the SL correction to the two-jet rate can be expressed in closed form as a function of x:

$$G^{(2,{\rm SL})}(x,T) = \frac{1-x^{\bar{\alpha}_{\rm S}T}}{x\ln\left(\frac{1}{x}\right)}\,\mathcal{G}^{(2,{\rm SL})}\left[\bar{\alpha}_{\rm S}\ln\left(\frac{1}{x}\right)\right], \tag{3.30}$$

where

$$\mathcal{G}^{(2,{\rm SL})}[z] = \ln\Gamma(1-z) - \ln\Gamma(1+z) - 2\gamma_{\rm E} z\,. \tag{3.31}$$

Notice that the expression (3.30) is singular at $\bar{\alpha}_{\rm S}\ln(1/x)=1$. From the viewpoint of the Mellin transformation (2.3), it is this singularity that produces the divergence of the series in ω-space. Conversely, use of the more correct expression (3.17) in ω-space, with the kernel function K_ω given in eq. (3.15), should suffice to remove the singularity in x-space.

3.3 Three-jet rate

The method used above for the two-jet rate can be extended, albeit laboriously, to higher jet multiplicities. In the three-jet case we have

$$R^{(3\,{\rm jet})}_\omega(Q^2,\mu_{\rm R}^2) = \frac{F^{(3\,{\rm jet})}_\omega(Q^2,\mu_{\rm R}^2,\mu^2)}{F_\omega(Q^2,\mu^2)} = \exp\left[-\gamma_{\rm L}\left(\bar{\alpha}_{\rm S}/\omega\right)T\right]\, G^{(3)}_\omega(T)\,, \tag{3.32}$$

where

$$\begin{aligned} G^{(3)}_\omega(T) = \int_{\mu_{\rm R}^2}^{Q^2} & \frac{dq_j^2}{q_j^2}\frac{dq_{j'}^2}{q_{j'}^2}\frac{dq_{j''}^2}{q_{j''}^2}\frac{d\phi_{j'}}{2\pi}\frac{d\phi_{j''}}{2\pi}K_\omega(q_j^2,\mu_{\rm R}^2)\times \\ & \times K_\omega(|q_j+q_{j'}|^2,\mu_{\rm R}^2)K_\omega(|q_j+q_{j'}+q_{j''}|^2,\mu_{\rm R}^2)\,. \end{aligned} \tag{3.33}$$

One could in principle evaluate this expression numerically using eq. (3.15) for K_ω. Here we derive the perturbative expansion analogous to eq. (3.22). Introducing $t=\ln(q_j^2/\mu_{\rm R}^2)$ etc. for brevity, the results in the appendix give

$$G^{(3)}_\omega(T) = \int_0^T dt\, H_\omega(t)\left[\int_0^T dt'\, L_\omega\left(\max\{t,t'\}\right) + 2\sum_{m=1}^\infty \zeta(2m+1)\,\frac{\partial^{2m}L_\omega}{\partial t^{2m}}\right], \tag{3.34}$$

where we write $H_\omega(q_j^2,\mu_{\rm R}^2)$ as $H_\omega(t)$ and

$$L_\omega(t) = H_\omega(t)[\ln H_\omega(t) - \ln H_\omega(T)] + t\,[H_\omega(t)]^2 + 2\sum_{m'=1}^\infty (2m')!\,\zeta(2m'+1)\,[H_\omega(t)]^{2m'+2}\,. \tag{3.35}$$

Hence we obtain

$$G_\omega^{(3)}(T) = G_\omega^{(3,\mathrm{DL})}(T) + G_\omega^{(3,\mathrm{SL})}(T)\,, \tag{3.36}$$

where $G_\omega^{(3,\mathrm{DL})}$ is the double-logarithmic result [6]

$$\begin{aligned} G_\omega^{(3,\mathrm{DL})}(T) = {}& \frac{1}{6}\ln^3\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right) + \ln^2\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right) + \\ & + \ln\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)^{-1} - \frac{\bar\alpha_{\mathrm{S}}}{\omega}T\left(1+\frac{3\bar\alpha_{\mathrm{S}}}{2\omega}T\right)\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)^{-2} \end{aligned} \tag{3.37}$$

and

$$\begin{aligned} G_\omega^{(3,\mathrm{SL})}(T) = {}& -2\sum_{m=1}^{\infty}(2m)!\,\zeta(2m+1)\left(\frac{\bar\alpha_{\mathrm{S}}}{\omega}\right)^{2m+1}\left\{\left[1-\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)^{-2m-2}\right] - \right. \\ & \left. -\frac{1}{2m+1}\left[\ln\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)+\psi(2m+1)+\gamma_{\mathrm{E}}+2\right]\left[1-\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)^{-2m-1}\right]\right\} + \\ & + 4\sum_{m,m'=1}^{\infty}\frac{(2m+2m'+1)!}{(2m+2m'+2)(2m'+1)}\,\zeta(2m+1)\,\zeta(2m'+1)\left(\frac{\bar\alpha_{\mathrm{S}}}{\omega}\right)^{2m+2m'+2}\times \\ & \times\left[1-\left(1+\frac{\bar\alpha_{\mathrm{S}}}{\omega}T\right)^{-2m-2m'-2}\right]. \end{aligned} \tag{3.38}$$

The expansion in eq. (3.38) is again strongly divergent for all values of $\bar\alpha_{\mathrm{S}}/\omega$. As discussed in subsect. 3.2, it can still be interpreted as an asymptotic expansion and used as a guide to the order of magnitude of the SL correction. Furthermore, the divergence is cured upon inverting the Mellin transformation to obtain the jet rate in x-space, as long as $\bar\alpha_{\mathrm{S}}\ln(1/x) < 1$. To extend the prediction to smaller values of x one would need to evaluate eq. (3.33) numerically using the full kernel function in eq. (3.15).

4. Generating function for multi-jet rates

For a general jet multiplicity r, we can write

$$R_\omega^{(r\,\mathrm{jet})}(Q^2,\mu_{\mathrm{R}}^2) = \frac{1}{r!}\,\frac{\partial^r}{\partial u^r}R_\omega(u,T)\bigg|_{u=0}\,, \tag{4.1}$$

where the jet-rate generating function R_ω is given by

$$R_\omega(u,T) = \exp[-\gamma_{\mathrm{L}}\,(\bar\alpha_{\mathrm{S}}/\omega)\,T]\,G_\omega(u,T) \tag{4.2}$$

and

$$G_\omega(u,T) = \sum_{r=0}^{\infty}u^r G_\omega^{(r)}(T)\,. \tag{4.3}$$

The function $G_\omega^{(r)}(T)$ was given in eqs. (3.6), (3.21) and (3.36) for $r = 1, 2$ and 3, respectively.

We can obtain the perturbative expansion of the function $G_\omega(u,T)$ as follows. We first define the unintegrated function $g_\omega(u,t,T)$ such that

$$G_\omega(u,T) = 1 + \int_0^T dt\, g_\omega(u,t,T)\,. \tag{4.4}$$

Then using the results in the appendix we find that $g_\omega(u,t,T)$ satisfies the integro-differential equation

$$g_\omega(u,t,T) = uH_\omega(t)\left[1 + \int_0^T dt'\, g_\omega\left(u, \max\{t,t'\}, T\right) + 2\sum_{m=1}^{\infty}\zeta(2m+1)\,\frac{\partial^{2m} g_\omega}{\partial t^{2m}}\right]. \tag{4.5}$$

Writing

$$g_\omega(u,t,T) = \sum_{n=0}^{\infty} c_n(u,t,T)\left(\frac{\bar{\alpha}_{\rm S}}{\omega}\right)^n\,, \tag{4.6}$$

this implies that

$$\begin{aligned} c_{n+1}(u,t,T) = u\,\delta_{n,0} - (1-u)t\, c_n(u,t,T) + u\int_t^T dt'\, c_n(u,t',T) + \\ +2u\sum_{m=1}^{\infty}\zeta(2m+1)\frac{\partial^{2m} c_n}{\partial t^{2m}}\,. \end{aligned} \tag{4.7}$$

Starting from $c_0 = 0$, this gives c_n iteratively as a polynomial in u, t and T, which can be substituted in eq. (4.4) to obtain the perturbative expansion of $G_\omega(u,T)$ to any desired order.[3] The relation (3.29) can then be used to transform the result directly to x-space, giving

$$F^{(r\,{\rm jet})}(x,Q^2,\mu_{\rm R}^2) = \frac{1}{r!}F(x,\mu_{\rm R}^2)\otimes\left.\frac{\partial^r}{\partial u^r}G(u,x,T)\right|_{u=0}\,, \tag{4.8}$$

where

$$G(u,x,T) = \delta(1-x) + \int_0^T dt\, g(u,x,t,T)\,, \tag{4.9}$$

with

$$g(u,x,t,T) = \frac{\bar{\alpha}_{\rm S}}{x}\sum_{n=0}^{\infty} c_{n+1}(u,t,T)\frac{\left[\bar{\alpha}_{\rm S}\ln\left(\frac{1}{x}\right)\right]^n}{n!}\,, \tag{4.10}$$

which we believe to be a convergent series as long as $\bar{\alpha}_{\rm S}\ln(1/x) < 1$.

4.1 Anomalous dimension

Notice that for $u = 1$ we have

$$R_\omega(1,T) = \sum_{r=0}^{\infty} R_\omega^{(r\,{\rm jet})}(Q^2,\mu_{\rm R}^2) = 1 \tag{4.11}$$

[3] The results agree with those given to fourth order in ref. [9].

and therefore

$$G_\omega(1,T) = \exp\left[\gamma_{\rm L}\left(\bar\alpha_{\rm S}/\omega\right)T\right]. \tag{4.12}$$

To show that eq. (4.5) does indeed lead to the Lipatov result (2.10) for the anomalous dimension, we note that when $u = 1$ the solution of eq. (4.5) is

$$\begin{aligned} g_\omega(1,t,T) &= \gamma\, e^{\gamma(T-t)} \\ G_\omega(1,T) &= 1 + \int_0^T dt\, g_\omega(1,t,T) = e^{\gamma T}\,, \end{aligned} \tag{4.13}$$

where

$$\begin{aligned} \gamma &= H_\omega(t)\left[1 + \gamma t + 2\sum_{m=1}^{\infty}\zeta(2m+1)\,\gamma^{2m+1}\right] \\ &= \frac{\bar\alpha_{\rm S}}{\omega}\left[1 + 2\sum_{m=1}^{\infty}\zeta(2m+1)\,\gamma^{2m+1}\right] \\ &= \frac{\bar\alpha_{\rm S}}{\omega}\gamma\left[\frac{1}{\gamma} - 2\gamma_{\rm E} - \psi(1+\gamma) - \psi(1-\gamma)\right]. \end{aligned} \tag{4.14}$$

Rearranging terms, we obtain $\gamma = \gamma_{\rm L}\left(\bar\alpha_{\rm S}/\omega\right)$ given by eqs. (2.10) and (2.11).

4.2 Jet multiplicity moments

We can compute the moments of the jet multiplicity distribution by successively differentiating the generating function at $u = 1$:

$$\langle r(r-1)\dots(r-s+1)\rangle = \left.\frac{\partial^s}{\partial u^s}R_\omega(u,T)\right|_{u=1}. \tag{4.15}$$

In this way we obtain the perturbative expansion of the mean number of jets

$$\begin{aligned} \langle r\rangle &= T\frac{\bar\alpha_{\rm S}}{\omega} + \frac{1}{2}T^2\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^2 + 2\zeta(3)T\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^4 + \\ &\quad + 4\zeta(3)T^2\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^5 - 8\zeta(5)T\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^6 + \cdots \end{aligned} \tag{4.16}$$

and the mean square fluctuation in this number,

$$\begin{aligned} \langle r^2\rangle - \langle r\rangle^2 &= T\frac{\bar\alpha_{\rm S}}{\omega} + \frac{3}{2}T^2\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^2 + \frac{2}{3}T^3\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^3 - 2\zeta(3)T\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^4 + \\ &\quad + 12\zeta(3)T^2\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^5 - \left(8\zeta(5)T - \frac{40}{3}\zeta(3)T^3\right)\left(\frac{\bar\alpha_{\rm S}}{\omega}\right)^6 + \cdots \end{aligned} \tag{4.17}$$

It appears true to all orders to SL precision that, as in the DL approximation [6], the mean number of jets is a quadratic function of T and the mean square fluctuation is a cubic function of T. Thus the distribution of jet multiplicity at small x and large T is narrow, in the sense that its r.m.s. width increases less rapidly than its mean as T increases.

5. Conclusions

The calculation of jet rates at small x poses many interesting challenges and sheds new light on the novel dynamics of this kinematic region. In the present paper we have concentrated on those perturbative contributions which have a factor of $\ln(1/x)$ for each power of $\alpha_{\rm S}$ and are further enhanced by one or more powers of $T = \ln(Q^2/\mu_{\rm R}^2)$, $\mu_{\rm R}$ being the minimum resolved jet transverse momentum.

For sufficiently large values of $\mu_{\rm R}^2$ and $Q^2 \gg \mu_{\rm R}^2$, the resummation of such terms would seems to be a well-defined problem in perturbation theory. The results in sect. 4 do indeed specify all terms of the form $(\bar\alpha_{\rm S}/\omega)^n T^m$ with $m > 0$, for any jet multiplicity, ω being the moment variable in the Mellin transform. However, as we have seen explicitly for the two- and three-jet rates (and we believe to be true more generally), the single-logarithmic terms (those with $0 < m < n$) cannot be resummed directly since they form strongly divergent series. In ω-space, the divergence is associated with kinematic regions in which the vector sums of transverse momenta of combinations of jets are less than $\mu_{\rm R}$. A more careful treatment of such regions renders the jet rates well-defined as integrals. Furthermore, one obtains convergent series, within a limited range of x, after performing the inverse Mellin transformation to x-space. In the case of the two-jet rate, we were able to sum the resulting series explicitly, to obtain a closed-form expression valid in the region $\bar\alpha_{\rm S}\ln(1/x) < 1$.

A number of interesting questions arise from our results. Clearly one would like to extend the resummation of jet rates to higher multiplicities and smaller values of x. This will require an x-space treatment of the difficult kinematic regions mentioned above. One would also like to prove the conjectures in subsect. 4.2 about jet multiplicity moments to all orders, and preferably to resum them. Ultimately, next-to-leading terms in $\ln(1/x)$ should also be included. Such terms will arise from next-to-leading corrections to the BFKL kernel and from the resolution of emitted gluons into two jets.

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A. Useful integrals

To evaluate integrals of the form

$$\int_{\mu^2}^{Q^2} \frac{dq^2}{q^2}\frac{d\phi}{2\pi} f(|q+k|^2) \tag{A.1}$$

to logarithmic accuracy, we may assume that $f(k^2)$ has an expansion in powers of $\ln k^2$, and so it suffices to consider $f(k^2) = \ln^p k^2$ for positive integer values of p.

This can in turn be done by considering $f(k^2) = k^{2\nu}$ for small ν and extracting the coefficient of $\nu^p/p!$ Now

$$\int_{\mu^2}^{Q^2} \frac{dq^2}{q^2}\frac{d\phi}{2\pi}|q+k|^{2\nu} = k^{2\nu}\int_{\mu^2}^{Q^2}\frac{dq^2}{q^2}\frac{d\phi}{2\pi}\left(1+\frac{q}{k}e^{i\phi}\right)^{\nu}\left(1+\frac{q}{k}e^{-i\phi}\right)^{\nu} \tag{A.2}$$
$$= k^{2\nu}\sum_{m=0}^{\infty}\binom{\nu}{m}^2\left[\int_{\mu^2}^{k^2}\frac{dq^2}{q^2}\left(\frac{q^2}{k^2}\right)^m + \int_{k^2}^{Q^2}\frac{dq^2}{q^2}\left(\frac{q^2}{k^2}\right)^{\nu-m}\right].$$

If $k^2 < \mu^2$ the first integral on the right-hand side is absent and the lower limit on the second becomes μ^2; if $k^2 > Q^2$ the second integral is absent and the upper limit on the first becomes Q^2. Thus, neglecting power-suppressed terms, we have

$$\begin{aligned}\int_{\mu^2}^{Q^2}\frac{dq^2}{q^2}\frac{d\phi}{2\pi}|q+k|^{2\nu} &= \frac{1}{\nu}\left(Q^{2\nu}-\mu^{2\nu}\right) && \text{for } k^2<\mu^2,\\ &= k^{2\nu}\Bigg[\ln\left(\frac{k^2}{\mu^2}\right)+\frac{1}{\nu}\left(\frac{Q^2}{k^2}\right)^{\nu}-\frac{1}{\nu}-\\ &\qquad -2\gamma_{\rm E}-\psi(1+\nu)-\psi(1-\nu)\Bigg] && \text{for } \mu^2<k^2<Q^2,\\ &= k^{2\nu}\ln\left(\frac{Q^2}{\mu^2}\right) && \text{for } k^2>Q^2.\end{aligned} \tag{A.3}$$

Here we have used the remarkable result, valid for Re $\nu > -1$,

$$\sum_{m=1}^{\infty}\binom{\nu}{m}^2\left[\frac{1}{m}+\frac{1}{m-\nu}\right] = 2\sum_{m=1}^{\infty}\zeta(2m+1)\,\nu^{2m} = -2\gamma_{\rm E}-\psi(1+\nu)-\psi(1-\nu)\,. \tag{A.4}$$

Thus, neglecting power-suppressed terms, we find

$$\begin{aligned}\int_{\mu^2}^{Q^2}\frac{dq^2}{q^2}\frac{d\phi}{2\pi}f(|q+k|^2) &= \int_{\mu^2}^{Q^2}\frac{dq^2}{q^2}f(q^2) && \text{for } k^2<\mu^2,\\ &= \int_{\mu^2}^{Q^2}\frac{dq^2}{q^2}f(\max\{k^2,q^2\})+\\ &\quad +2\sum_{m=1}^{\infty}\zeta(2m+1)\left(k^2\frac{\partial}{\partial k^2}\right)^{2m}f(k^2) && \text{for } \mu^2<k^2<Q^2,\\ &= \ln\left(\frac{Q^2}{\mu^2}\right)f(k^2) && \text{for } k^2>Q^2.\end{aligned} \tag{A.5}$$

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